

Rotation and shift of a camera with thin lens --
in reaction to focal plane shutter operation

Out[2]=

Center of gravity:

In[3]:=

$$\text{CoG} = m_0 r_0 + m_1 r_1 - m_2 r_2 == 0$$

Out[3]=

$$m_0 r_0 + m_1 r_1 - m_2 r_2 == 0$$

Out[4]=

m_0 shutter mass, m_1 body mass, m_2 lens mass idealized
point masses. r_0 , r_1 and r_2 the distance to
the center of gravity of respective masses

Out[5]=

Moments of inertia:

In[6]:=

$$J_0 = m_0 r_0^2$$

Out[6]=

$$m_0 r_0^2$$

In[7]:=

$$J_1 = m_1 r_1^2$$

Out[7]=

$$m_1 r_1^2$$

In[8]:=

$$J_2 = m_2 r_2^2$$

Out[8]=

$$m_2 r_2^2$$

In[9]:=

Out[9]=

Preservation of rotational
momentum when shutter operates:

In[10]=

$$\text{PRM} = -J_0 p_0 + J_1 p_1 + J_2 p_1 = 0$$

Out[10]=

$$-m_0 p_0 r_0^2 + m_1 p_1 r_1^2 + m_2 p_1 r_2^2 = 0$$

Out[11]=

where p_0 is the angle of rotation of the shutter (around CoG) and p_1 is the angle of rotation of body and lens (which rotate in common)

In[12]=

Out[12]=

Preservation of center of gravity (of linear momentum) in z direction (height):

In[19]=

$$h = s (m_0 + m_1 + m_2) - m_0 r_0 p_0 + m_1 r_1 p_1 - m_2 r_2 p_1 = 0$$

Out[19]=

$$-m_0 p_0 r_0 + m_1 p_1 r_1 - m_2 p_1 r_2 + (m_0 + m_1 + m_2) s = 0$$

Out[14]=

where s is the shift by which the CoG moves off the connection line between m_1 and m_2 , assuming the shutter mass m_0 initially was on that line and after operation, it rotated off that line by an angle p_0+p_1 . s is the amount by which the outer body+lens moves in height

In[15]=

Out[15]=

formula h applies to small rotation angles p only. Note that while p_1 is small, p_0 can exceed 10° . However, this induces a small error still.

In[43]=

Out[43]=

Approximate shutter move
in z direction y (note: $p1 \ll p0$):

In[32]=

$S_{move0} = y == r0 (p0 + p1)$

Out[32]=

$y = (p0 + p1) r0$

In[40]=

$S_{move1} = y == r0 p0$

Out[40]=

$y = p0 r0$

In[41]=

Out[41]=

And now solve for the shift s of body:

In[44]=

$Simplify[Solve[h /. Solve[CoG, r2], s] /.
Solve[S_{move0}, p0]] /. \{m0 + m1 + m2 \rightarrow M\}$

Out[44]=

$$\left\{ \left\{ \left\{ s \rightarrow \frac{m0 y}{M} \right\} \right\} \right\}$$

In[45]=

Out[45]=

Blur b due to rotation with focal length f:

In[46]=

$b = f p1$

Out[46]=

$f p1$

In[47]=

Out[47]=

thin lens focal length is sum
of distances of shutter and lens to CoG:

In[48]=

$$f = r0 + r2$$

Out[48]=

$$r0 + r2$$

In[58]=

brot =
Simplify[b /. Solve[PRM, p1] /. Solve[Smove1, p0] /.
Solve[CoG /. m0 → 0, r2]]

Out[58]=

$$\left\{ \left\{ \left\{ \frac{m0 r0 (m2 r0 + m1 r1) y}{m1 (m1 + m2) r1^2} \right\} \right\} \right\}$$

In[61]=

Out[61]=

Now, if the shutter is in
the center of gravity of the body:

In[62]=

Simplify[brot /. r0 → r1]

Out[62]=

$$\left\{ \left\{ \left\{ \frac{m0 y}{m1} \right\} \right\} \right\}$$

In[63]=

Out[63]=

Conclusion: rotational blur due to shutter shift
is always larger than the body translational
shift s and about independent on the weight or
focal length of the lens used. Surprising, eh?